Prove the derivative of $\sec x$ using the definition of the derivative function. **Show all steps.** SCORE: _____ / 5 PTS
Do NOT use the quotient rule, nor the known derivatives of any other trigonometric functions.
You may use the value of the two limits proved in lecture without proving them again.

$$\boxed{\lim_{h \to 0} \frac{\frac{1}{\cos(x+h)} - \frac{1}{\cos x}}{h}} \ \frac{1}{2}$$

$$= \boxed{\lim_{h \to 0} \frac{\cos x - \cos(x+h)}{h \cos(x+h) \cos x}} \ \frac{1}{2}$$

$$= \lim_{h \to 0} \frac{\cos x - \boxed{(\cos x \cosh - \sin x \sinh)}}{h \cos(x+h) \cos x} \ 1$$

$$= \lim_{h \to 0} \frac{\cos x(1 - \cosh) + \sin x \sinh}{h \cos(x+h) \cos x}$$

$$= \boxed{\lim_{h \to 0} -\frac{1}{\cos(x+h)} \cdot \lim_{h \to 0} \frac{\cosh - 1}{h}} \ \frac{1}{2}$$

$$\boxed{+ \lim_{h \to 0} \frac{\sin x}{\cos(x+h) \cos x} \cdot \lim_{h \to 0} \frac{\sinh}{h}} \ \frac{1}{2}$$

$$= \underbrace{-\frac{1}{\cos x} \cdot 0}_{\frac{1}{2}} + \underbrace{\frac{\sin x}{\cos^2 x}}_{\frac{1}{2}} \cdot \underbrace{1}_{\frac{1}{2}}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \underline{\sec x \tan x} \ \frac{1}{2}$$

The position of an object at time t is given by $s(t) = (t^3 - 5t)3^t$. **SCORE: _____ / 5 PTS**

Find the **acceleration** of the object at time $t = 2$.

1 1

$$s'(t) = (t^3-5t)'3^t + (t^3-5t)(3^t)' = \boxed{(3t^2-5)3^t} + \boxed{(t^3-5t)3^t\ln 3}$$

$$s''(t) = (3t^2-5)'3^t + (3t^2-5)(3^t)' + (t^3-5t)'3^t\ln 3 + (t^3-5t)(3^t\ln 3)'$$

$$= \boxed{6t\cdot 3^t} + \boxed{(3t^2-5)(3^t\ln 3)} + \boxed{(3t^2-5)(3^t\ln 3)} + \boxed{(t^3-5t)(3^t(\ln 3)^2)}$$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$$s''(2) = 12\cdot 9 + 7\cdot 9\ln 3 + 7\cdot 9\ln 3 + (-2)\cdot 9(\ln 3)^2$$

$$= \boxed{108 + 126\ln 3 - 18(\ln 3)^2}$$ 1

Find the slope of the tangent line to the curve $y = \dfrac{f(x)}{x^2}$ at the point where $x = 3$ if $f(3) = -4$ and $f'(3) = 2$. **SCORE: _____ / 4 PTS**

$$\frac{dy}{dx} = \frac{f'(x)\,x^2 - f(x)(x^2)'}{(x^2)^2} = \boxed{\frac{x^2 f'(x) - 2x f(x)}{x^4}} \quad 2$$

$$\left.\frac{dy}{dx}\right|_{x=3} = \frac{3^2 \cdot f'(3) - 2(3)\cdot f(3)}{3^4} = \boxed{\frac{9\cdot 2 - 6\cdot(-4)}{81}} = \frac{42}{81} = \boxed{\frac{14}{27}}$$

$1\frac{1}{2}$ $\qquad$ $\frac{1}{2}$

$$\frac{d}{dy}(5y^e + 7e^y - 3\pi^e)$$

$= \underbrace{5ey^{e-1}}_{1} + \underbrace{7e^y}_{1}$

$+\frac{1}{2}$ FOR NOT WRITING A DERIVATIVE FOR $-3\pi^e$ (IE. DERIVATIVE $= 0$)

$$\frac{d}{dx}\,\frac{2-3x^2+x^4}{1-2x^4}$$ (Your final answer must be one fraction)

$$=\frac{\boxed{(-6x+4x^3)(1-2x^4)-(2-3x^2+x^4)(-8x^3)}\ \ 2\tfrac{1}{2}}{\boxed{(1-2x^4)^2}\ \ \tfrac{1}{2}}$$

$$=\frac{-6x+12x^5+4x^3-\cancel{8x^7}+16x^3-24x^5+\cancel{8x^7}}{(1-2x^4)^2}$$

$$=\boxed{\frac{-12x^5+20x^3-6x}{(1-2x^4)^2}}\quad 1$$

$$\frac{d}{dt}(\csc t + \tan t \sec t)$$

$$= -\csc t \cot t + \sec^2 t \sec t + \tan t \sec t \tan t$$

$$= -\csc t \cot t + \sec^3 t \quad (1)$$

$$(1) \qquad + \sec t \tan^2 t \quad (1)$$

$$\frac{d^3}{dr^3}\frac{18r^2 - 27r}{4\sqrt[3]{r}}$$

$$= \frac{d^3}{dr^3}\left(\frac{9}{2}r^{\frac{5}{3}} - \frac{27}{4}r^{\frac{2}{3}}\right)$$

$$= \frac{d^2}{dr^2}\left(\boxed{\frac{15}{2}r^{\frac{2}{3}} - \frac{9}{2}r^{-\frac{1}{3}}}\right) \quad 1\tfrac{1}{2}$$

$$= \frac{d}{dr}\left(\boxed{5r^{-\frac{1}{3}} + \frac{3}{2}r^{-\frac{4}{3}}}\right) \quad 1$$

$$= \boxed{-\frac{5}{3}r^{-\frac{4}{3}} - 2r^{-\frac{7}{3}}} \quad 1$$

Prove the derivative of $\cot x$ using the quotient rule. **Show all steps.** **SCORE: _____ / 3 PTS**

You may use the known derivatives of *sin x* and *cos x* without proving them.

$$\cot x = \frac{\cos x}{\sin x}$$

$$(\cot x)' = \frac{(\cos x)' \sin x - \cos x (\sin x)'}{(\sin x)^2}$$

$$= \frac{\boxed{(-\sin x)\sin x - \cos x(\cos x)}\ 1\tfrac{1}{2}}{\boxed{(\sin x)^2}\ \tfrac{1}{2}}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= \boxed{-\frac{1}{\sin^2 x}}\ \tfrac{1}{2}$$

$$= \boxed{-\csc^2 x}\ \tfrac{1}{2}$$